

The simple linear regression model: $y = \beta_0 + \beta_1 x + u$. The OLS estimators are: $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$, $\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \sum_{i=1}^n w_i y_i$, where $w_i = \frac{x_i - \bar{x}}{\sum_{i=1}^n x_i (x_i - \bar{x})}$, $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$, $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$. Moreover, $Var(\hat{\beta}_0) = \frac{\sigma^2}{(n-1)n S_x^2} \sum_{i=1}^n x_i^2$, $Var(\hat{\beta}_1) = \frac{\sigma^2}{(n-1)S_x^2}$, where $S_x^2 = \frac{1}{(n-1)} \sum_{i=1}^n (x_i - \bar{x})^2$.

The multiple linear regression model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + u$. Interpretation of β_j : holding x_i , $i = 1, \dots, k$ and $i \neq j$, fixed implies that $\Delta \hat{y} = \hat{\beta}_j \Delta x_j$, $j = 1, \dots, k$. Moreover, $Var(\hat{\beta}_j) = \frac{\sigma^2}{SST_j(1-R_j^2)}$, where the $SST_j = \sum_{i=1}^n (x_{ij} - \bar{x}_j)^2$ and R_j^2 is the R^2 from the regressing x_j on all other x 's. Moreover $\hat{\sigma}^2 = (\sum_{i=1}^n \hat{u}_i^2)/(n-k-1)$.

R-squared of regression: $R^2 = SSE/SST = 1 - SSR/SST = \frac{[\sum_{i=1}^n (\hat{y}_i - \bar{y})(y_i - \bar{y})]^2}{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2 \sum_{i=1}^n (y_i - \bar{y})^2}$, where $SST = \sum_{i=1}^n (y_i - \bar{y})^2$, $SSE = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$, $SSR = \sum_{i=1}^n \hat{u}_i^2$.

Suppose that we know that the true model is $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$, but we estimate $\tilde{y} = \tilde{\beta}_0 + \tilde{\beta}_1 x_1$. $E(\tilde{\beta}_1) = \beta_1 + \beta_2 Corr(x_1, x_2) \frac{S_{x_2}}{S_{x_1}}$, and the direction of bias is:

	$Corr(x_1, x_2) > 0$	$Corr(x_1, x_2) < 0$
$\beta_2 > 0$	Positive Bias	Negative Bias
$\beta_2 < 0$	Negative Bias	Positive Bias

Interpretation of partial effects:

y	x_j	$\Delta x_j = 1 \rightarrow \Delta E(y x) = \beta_j$
$\ln(y)$	x_j	$\Delta x_j = 1 \rightarrow \Delta E(y x) = 100\beta_j\%$
y	$\ln(x_j)$	$\Delta x_j = 1\% \rightarrow \Delta E(y x) = \beta_j/100$
$\ln(y)$	$\ln(x_j)$	$\Delta x_j = 1\% \rightarrow \Delta E(y x) = \beta_j\%$

Under the CLM assumptions, conditional on the sample values of the independent variables for $j = 0, \dots, k$, $\frac{\hat{\beta}_j - \beta_j}{sd(\hat{\beta}_j)} \sim N(0, 1)$, where $sd(\hat{\beta}_j) = \sqrt{Var(\hat{\beta}_j)}$; $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim t(n-k-1)$, where $se(\hat{\beta}_j) = \frac{\hat{\sigma}}{\sqrt{SST_j(1-R_j^2)}}$.

A $100(1-\alpha)\%$ CI: $(\hat{\beta}_j - t_{\alpha/2} se(\hat{\beta}_j), \hat{\beta}_j + t_{\alpha/2} se(\hat{\beta}_j))$.

$$F = \frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-k-1)} = \frac{(R_{ur}^2 - R_r^2)/q}{(1 - R_{ur}^2)/(n-k-1)}$$

Let $\widehat{\log(y)} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k$, then $\hat{y} = \exp(\frac{\hat{\sigma}^2}{2}) \exp(\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k)$.